Intelligent Systems: Reasoning and Recognition

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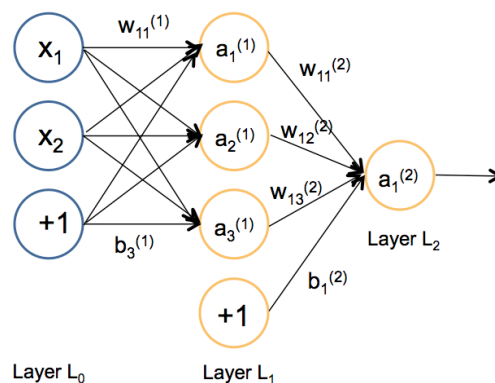
Artificial Neural Networks.

The following by may be performed by computer program or by hand. The recommended method is to write an interactive Python program.

Assume that you have a 2-layer network with 3 hidden units, and one output, using a sigmoid activation function.

The weights for the first layer have been initialized to the following (random) parameters:

$$W_{ji}^{(1)} = \begin{pmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \end{pmatrix}, \; b_j^1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \; W_{ji}^{(2)} = \begin{pmatrix} 1 & -1 & 0 \end{pmatrix} \text{ and } b_j^2 = \begin{pmatrix} 0 \end{pmatrix}$$

You have the following 4 samples of training data.

$$\{\vec{X}_m\} = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\} \text{ with } \{y_m\} = \{1,0,1,0\}$$

a) Calculate the output of each unit $a_{ji}^{(l)}$ for the first training sample $\vec{X}_1$.

b) Calculate the error term $\delta_{j,1}^{(l)}$ for each unit by back-propagation for the first training sample $\vec{X}_1$.

c) Calculate the corrections $\Delta w_{ji,1}^{(l)}$ and $\Delta b_{j,1}^{(l)}$ for the first training sample $\vec{X}_1$.

d) Apply the correction to the network parameters for layer 2 using a learning rate of $\eta=0.5$.

e) (optional) Write a python code to evaluate $\Delta w_{ji,m}^{(l)} = a_i^{(l-1)} \delta_{j,m}^{(l)}$ and $\Delta b_{j,m}^{(l)} = \delta_{j,m}^{(l)}$ for all 4 training sample $\{\vec{X}_m\}$ and $\{y_m\}$ without applying the correction to the weights. Compute the average of the correction factors and then update the weights with a learning rate of $\eta=0.5$ using the average. What are the resulting network parameters?